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## Abstract

LLC and LCC resonance converters are discussed and compared in this paper. We start with a retrospect of recent development and a discussion of resonance versus pulse width modulated converters.

The behaviour of resonance converters can be analysed by the traditional Fundamental Harmonic Approach (FHA) or a newer Time Domain Approach, developed by the author. The FHA is an approximation, the Time Domain Approach is not, and it gives the user a fast and clear overview over operating data, component stress, transformer magnetization and power loss etc. which are not provided by any other known design tool, including simulation. The Time domain Approach also calculates oscilloscope pictures of currents and voltages at any working point. The power of the Time Domain Approach is demonstrated by examples of LLC and LCC calculations, showing some but not all of its capabilities.

Severe non-linearity control problems for the LCC and the LLC converters are explained, and Charge Mode Control is shown to be the universal solution to them. Charge Mode Control turns the power stage into a controlled power source.

## Introduction to resonant power converters

For many years, the phrase "resonance converters" was used among electronics engineers and management people as a buzz word for the ideal way to do electronic power conversion.

Already at my own introduction to switch mode power supplies (SMPS) in 1978 there was work going on at universities and large electronics companies, dealing with a new way of power conversion by means of topologies containing resonant LC circuits.

The drawbacks of the more conventional pulse width modulated SMPS techniques were quite well understood. Compared to the ideal situation, there were always a number of unwanted side effects in the so-called "hard switched" converter topologies:

- Significant switching losses due to parasitic inductances and capacitances and due to the limited speed of semiconductor switches.
- Snubbers (clamping circuits) usually have to be incorporated to limit unwanted and dangerous voltage and current spikes. Nearly all known snubber networks add more power losses.
- Generation of problematic Electro Magnetic Interference caused by the high frequency of operation, and especially by the unlimited steepness of voltage and current slopes at the switching instants.

These side effects were – and are still today – an obstacle when trying to build a compact and low-noise power supply: compactness requires a high switching frequency, thereby power losses become higher, efficiency becomes worse, while temperature and electromagnetic noise increase.

The "resonance converter" ideas were born as an attempt to overcome these drawbacks. By adding resonance L+C components to the switching circuit, it may be possible to eliminate switching losses and create more soft voltage and current transitions, thereby improving efficiency and reducing noise. Part of the idea may be to let a leakage inductance or a parasitic capacitance work *for* you, not *against* you as with hard switching.

Still today, resonance converters are some times spoken about as the ultimate way to build a good power supply, especially among management people.

However, if we look around, most switch mode power supplies are still conventional hard switched types like the buck and the boost, the single- or double switch forward, the flyback or the half -/ full bridge forward converters.

Why does nearly nobody use a resonance converter? This seems to be a good question.

First it should be mentioned that semiconductor switches like mosfets and diodes have become many times better and faster, since 1978. This, of course, has lead to a large increase of the average switching frequency, so that SMPS today can be made much more compact and elegant than ever before. The switching frequency has followed the speed of the semiconductors, so the problem of switching loss has not really changed. The issues of emitted noise has even become more challenging than before.

So the arguments for the resonance converter still seem to be valid.

Next, we should address the fact that a resonance converter is not just one thing. A resonance converter belongs to an inexhaustible world of ideas and topologies, compared to which the hard switched world seems to be much easier to grasp. There are different classes of resonance converters, e.g. "true" resonance converters, quasi resonance converters and many other types of soft switching converter ideas. Many of these converter types can

even be operated in two or more modes, so the diversity of resonance converters and the ways to make them work is enormous.

It seems to be true, that most of the resonance converter ideas, brought about during the last decades, have too large drawbacks in common applications, to justify their nice properties. As an example rms current or peak voltages are significantly higher in quasi resonant topologies than in their hard switched counterparts.

Another important difference is, that for conventional converter topologies it is possible to set up a relatively simple mathematical description of their behaviour, e.g. a static transfer function, so that you don't have to be an Einstein to design a good SMPS.

For resonance converters it is different. Transfer functions are usually strongly non-linear, and they are beyond the area of simple mathematical descriptions or comprehension by human mind. Most often, the best design tool is a simulation tool like Pspice, which is really not good enough. It does not give you much insight in the overall behaviour of a converter, and it does not provide a fast and effective optimization.

Usually, a power converter involves a feedback system to maintain e.g. a fixed output voltage or current. When designing a feedback loop, a non-linear power transfer function is a very undesirable property, and the result may be an unusable compromise, which may disqualify the resonance converter in comparison to a conventional SMPS.

I think these reasons are enough to explain, why resonance converters are still so rarely seen: they are very difficult to comprehend and optimize, and most resonance ideas do have undesirable properties too. Most of those engineers who have tried to design one, have got their fingers burnt and have returned to the more well known topologies.

However, during the last 5 – 10 years a special type of resonance converter, the so-called LLC, has become still more popular, in particular in flat screen displays, TVs and other mass produced electronic devices, but they are not seen often in professional electronics. Lots of recent articles and application notes deal with the LLC, and many IC manufacturers have launched driver ICs for this topology. Indeed, the LLC can have a superior efficiency and low noise and a potentially very flat design, which is attractive in devices like modern TVs and displays.

## The LLC and the LCC converter

The most used LLC converter design is a half bridge, as shown in figure 1. Compared to the pulse width modulated half bridge, the LLC has an inductor  $L_s$  in the input and none in the output. The LLC also must have an inductor  $L$  in parallel to the transformer winding. Both inductors can be integrated in the transformer:  $L_s$  as a leakage inductance by physically separating primary and secondary winding,  $L$  as a magnetizing inductance by grinding an air gap in the ferrite. This makes two inductors + one transformer in one magnetic component. The two inductances, which are usually unwanted parasitics in a transformer, now become essential means to achieve soft and lossless switching.

The LLC has a sister called LCC. On a diagram they look like twins, the only difference being that in the LCC, the magnetizing inductance  $L$  has been replaced by a capacitor  $C_s$ . This capacitor can be located on either side but usually it is put on the secondary side.  $L_s$  can again be integrated in the transformer by physical separation of windings but  $C_s$  must be a discrete capacitor. An LCC transformer must not have an air gap in the core.

Both converters are resonance converters controlled by a variable frequency, and both are soft switching converters. But a closer study reveals two completely different ways of operation with completely different voltage and current waveforms and control properties.

The names "LLC" and "LCC" simply indicate that the LLC contains two inductors and one capacitor, and the LCC contains one inductor and two capacitors. The half bridge capacitors  $C_1$  and  $C_2$  are considered as one capacitor with the value  $C_1+C_2$ .

Traditional switching converters are controlled by pulse width or duty cycle, most of them at a fixed frequency. The LLC and the LCC – and most other resonance converters – are controlled by a variable frequency, the LLC and LCC always with 50% duty cycle.

Despite their apparent simplicity, the LLC and the LCC are not easy to analyse and optimize. They are some times referred to as “multiresonant” converters because they have two resonance frequencies: one when the output diodes are off, another resonance frequency when they are active. Both converters exhibit a peak “gain” at some apparent “resonance” frequency, however this frequency is not fixed but depends on load and conversion ratio. To maintain soft switching, both converters must be operated on the resonance slope above that “resonance” frequency. Power must be controlled by switching frequency: frequency up  $\Rightarrow$  power down.

But this simple statement is not enough to design these converters. It appears that the transfer functions are by no means linear, and for the LLC it appears that the “gain” of the transfer function can be close to infinity in typical operating conditions. These facts are not intuitive, not even in a qualitative sense, and since we also must be able to handle the converters in a quantitative sense, i.e. design them for a certain power, input and output voltage etc. we need a tool to help us. The engineering brain is far from powerful enough to solve this task.

### LLC converter analysis

Since the LLC is presently the most popular resonance converter, let us start with that. Figure 1 is transformed into the equivalent circuit in figure 3 to enable us to deal with the essential converter without the conversion ratio of a transformer involved. The bridge rectifier has been replaced by two diodes, each diode delivering current to its own output battery  $\pm V_o$ . The output voltage  $V_o$  is an imaginary voltage which will be related to the real output voltage  $V_{out}$  by the transformation ratio in the transformer.

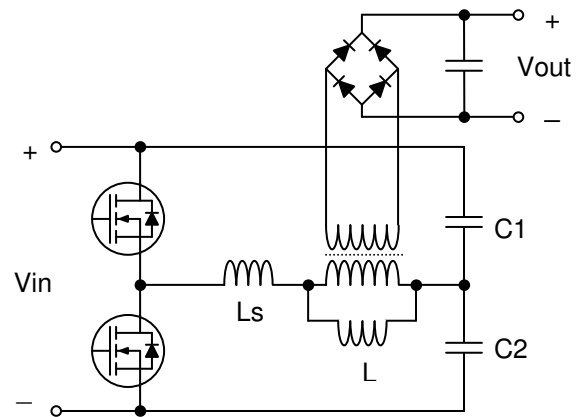


Figure 1 LLC

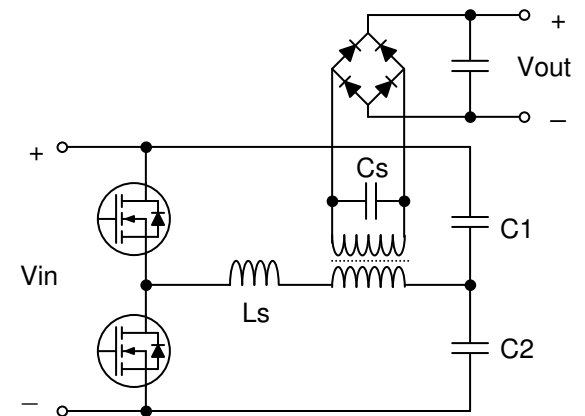


Figure 2 LCC

### Fundamental harmonic approach

The usual way to analyse this circuit is by means of the so-called Fundamental Harmonic Approach (FHA). The FHA is a linear approximation dealing only with the fundamental or first harmonic of voltage and current waveforms. It assumes that only the fundamental component of the input square wave contributes to power transfer. It is a reasonable approximation since currents are more or less sinusoidal because it is a resonance system. I think it is more fair to say that it is a necessary approximation because it has been the only practical way get results.

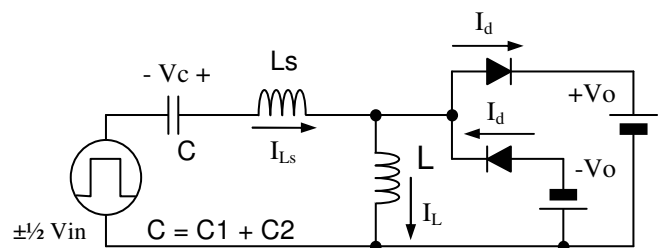


Figure 3 LLC equivalent circuit

The FHA transforms the non-linear circuit into an equivalent linear network, then it picks the fundamental harmonic of the input square wave and puts it to the input as seen in figure 4. On the output, the diode + load is replaced by a linear resistor with an equivalent value different from the real load resistance. Calculation of  $R_{ac}$  is based on the condition that the fundamental components of output voltage and current must dissipate the same power in  $R_{ac}$  as the real load power. The derivation of  $R_{ac}$  can be found in ref. 1 and 3.

Now we can use Laplace network theory to calculate the voltage gain in the frequency domain from input to output for different values of  $R_{ac}$ , i.e. for different loads.

The equations can be written very simple, for instance like the example in figure 5. There are two resonance frequencies:  $f_0$  and  $f_1$ . At high load (low  $R_{ac}$ ) the gain curves have a top at  $f_1$  which is the resonance between  $C$  and  $L_s$ . At low load the gain peak is higher and at very low load it peaks at  $f_0$ . Both frequencies are seen in the plots. The operating point must always be on the right hand slope of the actual curve, otherwise soft switching is

lost.

All curves pass through the “load independent point” at  $f_1$  with a gain of 1. This can be explained by the fact that  $f_1$  is the resonance between  $L_s$  and  $C$  which means they have zero impedance, so the input and output in figure 4 are connected.

The FHA is used in nearly all published papers and application notes on LLC design, among others in ref. 1, 2, 3, and 4. Most papers also try to guide you on how to manipulate the results of these graphs to convert them to a working LLC converter. Trying to follow the guides is hard work, and you very easily lose feeling of what you are doing. You cannot just read the operating data from the graphs, like for instance: What is the maximum possible power at a given input voltage? How does frequency move with power and input voltage? What is the rms current in the fets versus load and input voltage? What is peak-peak capacitor voltage versus load and input voltage?

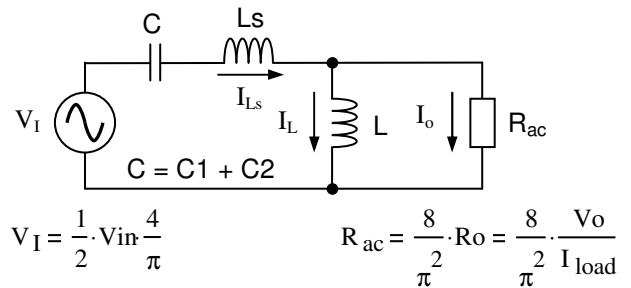


Figure 4 Equivalent linear AC model

**Selected component values:**

$C \equiv 2.47 \cdot 10^{-9}$

$L_s \equiv 184 \cdot 10^{-6}$

$L \equiv 900 \cdot 10^{-6}$

$\omega \equiv \frac{1}{\sqrt{(L_s + L) \cdot C}}$

$f_o := \frac{\omega}{2 \cdot \pi}$

$Z_s(\omega) := j \cdot \omega \cdot L_s + \frac{1}{j \cdot \omega \cdot C}$

$\omega_l := \frac{1}{\sqrt{L_s \cdot C}}$

$f_1 := \frac{\omega_l}{2 \cdot \pi}$

$Z_p(\omega, R_{ac}) := \frac{j \cdot \omega \cdot L \cdot R_{ac}}{j \cdot \omega \cdot L + R_{ac}}$

Gain equation:

$v_o(\omega, R_{ac}) := v_i \cdot \frac{Z_p(\omega, R_{ac})}{Z_s(\omega) + Z_p(\omega, R_{ac})}$

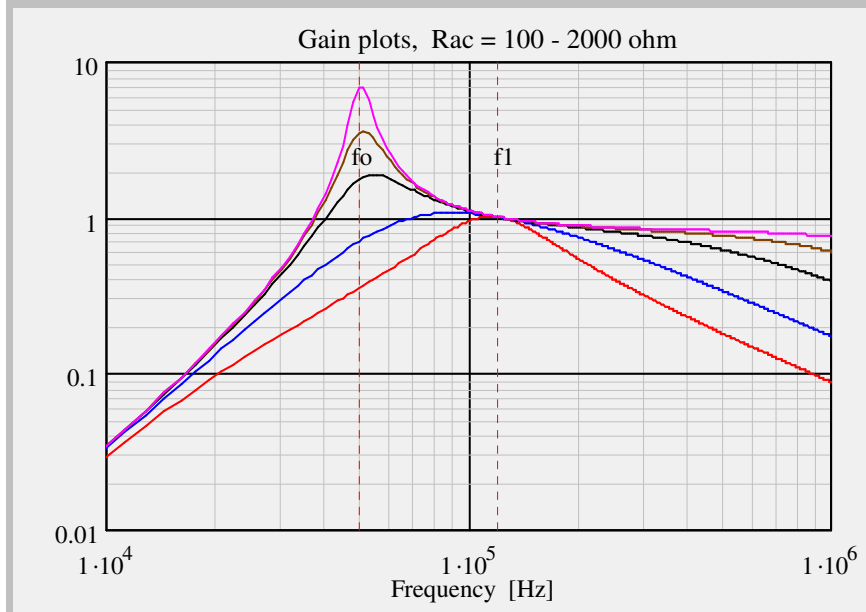


Figure 5 Fundamental Harmonic Approach  
 Gain versus frequency for  $R_{ac} = 2k\Omega, 1k\Omega, 500\Omega, 200\Omega, 100\Omega$

## The time domain approach

During my years as a self employed engineer I caught a special interest in the LCC converter in year 2000 but I soon realized that brainpower or Pspice simulation was not enough to design it. Probably, the FHA method could also have helped with the LCC but the FHA never really appealed to me.

In a period of low activity I therefore took the plunge into developing a mathcad tool using a Time Domain Approach (TDA). This approach works by finding time domain solutions to the time-varying state equations of the non-linear equivalent circuit in figure 3. Simply speaking, with a given set of L + C + C values, the TDA lets us calculate a scope picture of voltages and currents for each operating frequency at a selected input- and output voltage. The computer can extract a lot of data from each scope picture, for example rms currents, peak-peak voltages, etc. Then, by letting the computer run through a lot of scope pictures and extract data from them, it can eventually make plots of frequency versus power, rms currents versus power or whatever we want to see in an overview plot. Contrary to the FHA, these plots are 100% accurate, provided there are no losses in the circuit – a reasonable assumption since the reason for using a resonance converter is to reach a high efficiency.

Making a tool with a Time Domain Approach is not something you just do in a week. It requires a lot of time and a lot of dedication and perseverance. And also long time for debugging because you make errors all the time.

But the reward, when or if you succeed, is fabulous.

After another few years I was ready for the challenge of creating a similar analysis tool for the LLC converter which turned out to be much more complicated than the LCC. The LLC moves through two operational modes from zero load to peak load. The LLC will run through up to six useful operating modes, each mode requiring its own set of equations. For each mode, the design tool must solve the circuit's differential equations during one half switching cycle by using an iteration strategy. Furthermore, the tool must automatically detect when the circuit runs from one mode to the next mode, at which point it must use a new set of differential equations. A more detailed description of the calculation process is found in my homepage, ref. 8.

I will not recommend anyone in the industry to try it. It takes much more time than your boss will ever allow you to spend, and there is no guarantee for success. Most engineers will probably quit long before there is a useful result in sight. But I am a bit surprised that apparently not even a university has ever done it to the full, considering the usefulness of the TDA.

## Time domain approach for the LLC

In the next page you see an example with one complete cycle of calculated oscilloscope plots for the same LLC converter which was used for figure 5. In the first plots, the input voltage is below the Load Independent Point (LIP) which is  $400V (= 2 \cdot V_o)$ . In this range, the last part of each half cycle always has zero diode current.

In the lower plots, power is unchanged but input voltage is above the load independent point, in that case diode current typically lags behind the shift of input voltage.

Note the switching frequency printed in the current plots.

I think these plots are enough to make it clear why the LLC converter is so difficult to comprehend. This example even shows only two out of six possible operating modes. The six modes are described closer in ref. 8.

In figure 7 we see some results collected from a large amount of oscilloscope plots for this design, calculated at five different input voltages. To the left: switching frequency, to the right: rms currents in switches and in primary winding. The calculator also generates many other overview plots, among others primary peak-peak current, primary peak-peak voltage, resonance capacitor peak-peak voltage, input supply capacitor rms current, output capacitor rms current.

After insertion of transformer wire and core data it also shows you transformer magnetization, transformer wire loss on primary and secondary, and total estimated transformer loss (ref. 9).

In brief, you can see all relevant component stress data for a selected L+L+C design directly without having to do any manual manipulations first. And very important: the results are 100% accurate.

In figure 7 left part you see some important details. First, at the LIP (400V input), the switching frequency is completely constant, except at very low power where currents become discontinuous. Above the LIP, frequency is variable, especially at low load. Below LIP the curves are again practically horizontal in large ranges and the ends of the two lower curves show the theoretical maximum achievable power from this LLC converter. Not much in these curves look like familiar resonance graphs.

Also note that in this design, the frequency depends more on input voltage than on load.

Selected component values:

$C \equiv 2.47 \cdot 10^{-9}$      $L_s \equiv 184 \cdot 10^{-6}$      $L \equiv 900 \cdot 10^{-6}$

Fictitious output voltage in model:  $V_o \equiv 200$

Input voltage:  $V_{in} \equiv 350$

$I_L(t)$ : current in magnetizing inductor.  
 $I_{Ls}(t)$ : current in primary winding and switches.  
 $|I_d(t)|$ : current in output diodes all together (in transformerless model).

$V_i(t)$ : Input square wave voltage.  
 $V_C(t)$ : Resonance capacitor AC voltage.  
 $V_{prim}(t)$ : Voltage on input of resonance inductor  $L_s$ .  
 $V_L(t)$ : secondary voltage (in transformerless model). Will be clamped to  $\pm V_o$

Input voltage:  $V_{in} \equiv 430$

Soft switching occurs as long as the total current (blue trace) at the midpoint (dotted red line) is positive. Then, when the upper fet turns off, the current will automatically pull the midpoint voltage down to the negative rail.

At the transition to hard switching, the current waveforms are displaced more to the left so that the current comes back close to zero before switching. Below resonance (hard switching area) the current has already reversed at the switching instant, which means that when a fet turns off, current just continues to flow in its body diode. Nothing happens until the opposite fet turns on, at which instant the body diode will recover by force and very fast. This can generate a tremendous high frequency noise burst and it can be lethal to the fets.

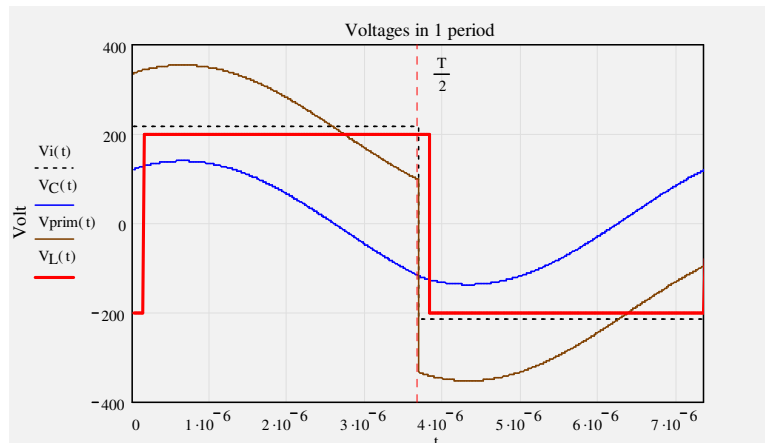
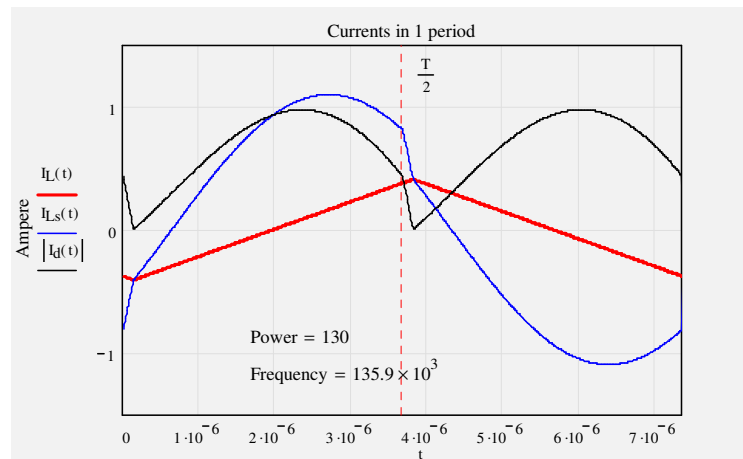
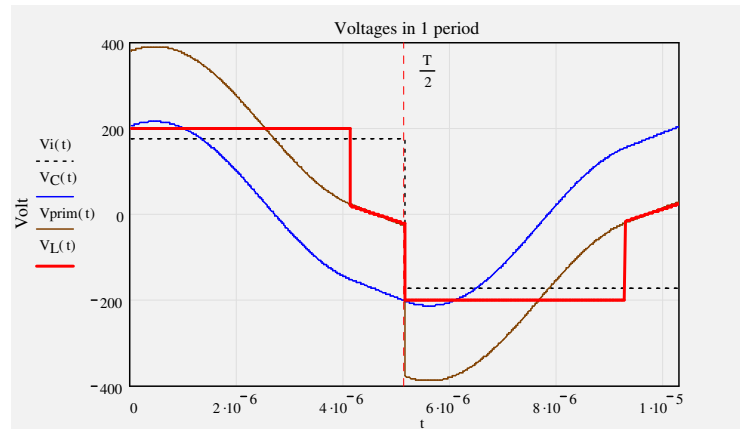
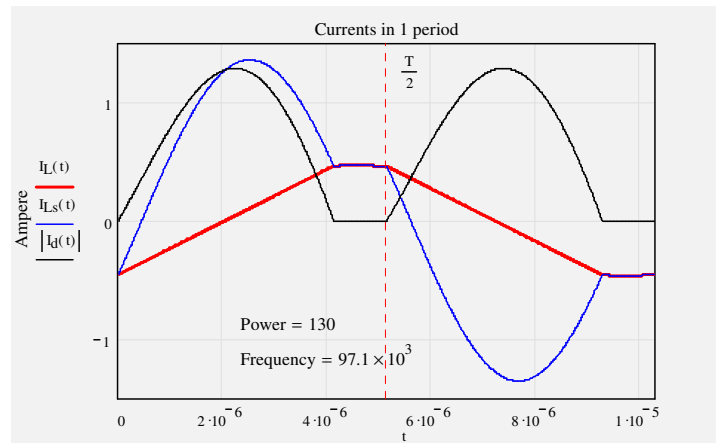
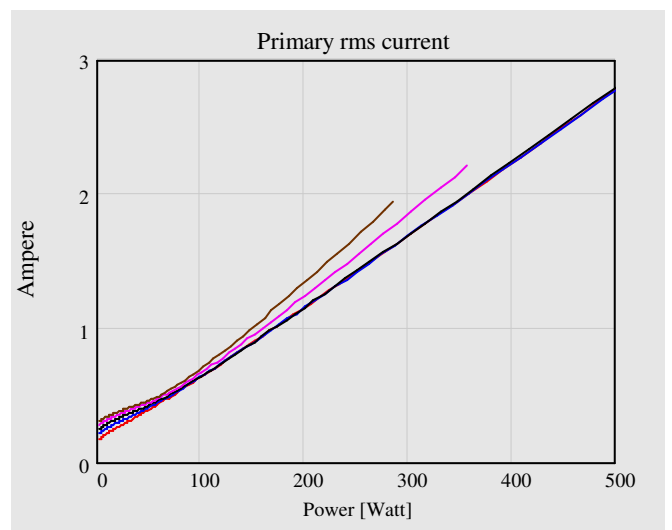
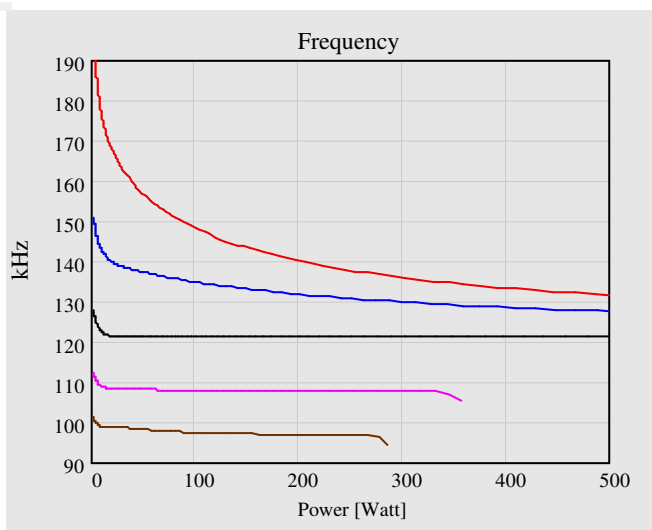


Figure 6 Calculated scope plots below and above LIP



Input voltages:

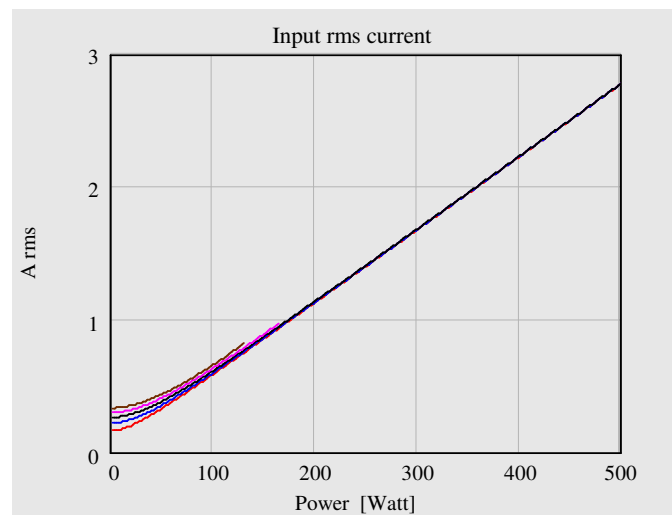
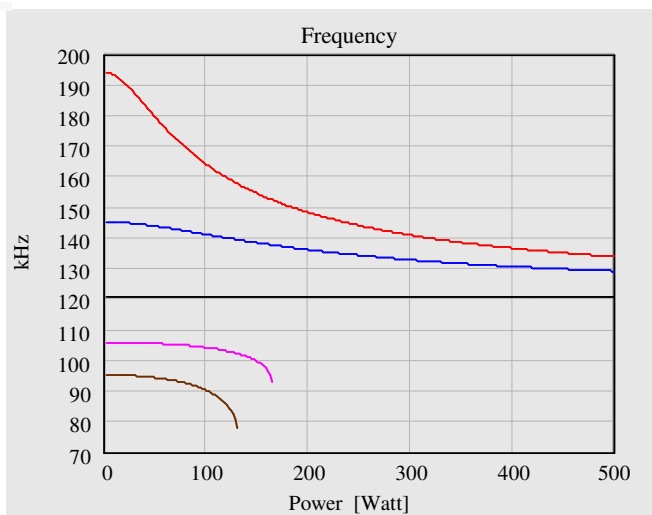
$$\begin{pmatrix} V_{I_4} \\ V_{I_3} \\ V_{I_2} \\ V_{I_1} \\ V_{I_0} \end{pmatrix} = \begin{pmatrix} 450 \\ 425 \\ 400 \\ 375 \\ 350 \end{pmatrix}$$

**Figure 7 LLC in Time Domain Approach**  
 Plots of frequency and input rms currents  
 at 5 input voltages

Now let us compare the results of the TDA with those of the FHA. Figure 5 is inappropriate for comparison but the results of figure 5 can be re-arranged into the same form as used in figure 7 for direct comparison (I wonder why nobody did this exercise). Figure 8 depicts this kind of FHA results for the LLC converter.

The most significant shortcoming of the FHA is the predicted max. achievable power below the LIP which is less than half of the real available power. Trusting only the FHA could lead to a severely oversized LLC converter.

I do not think this evaluation of the FHA method's accuracy has ever been possible before.



Input voltages:

$$\begin{pmatrix} V_{in_4} \\ V_{in_3} \\ V_{in_2} \\ V_{in_1} \\ V_{in_0} \end{pmatrix} = \begin{pmatrix} 450 \\ 425 \\ 400 \\ 375 \\ 350 \end{pmatrix}$$

**Figure 8 LCC in Fundamental Harmonic Approach**  
 Plots of frequency and input rms currents  
 at 5 input voltages



## The LCC converter

The basic LCC converter was shown in figure 2 which is repeated here.

For some unknown reason, the LCC converter is hardly ever used, and, opposed to the LLC, very few papers deal with how to design an LCC. I wonder why.

If you ask Google for LCC, it answers with LLC.

For some designs the LCC can do a better job than the LLC. About half of my recent resonance converter designs are LLC, the rest are LCC. I will tell more about the differences later.

It should be possible to set up a linear FHA model of the LCC, as we did for the LLC converter. I do not know if anybody did it and since I have the TDA tool to help me, it is not worth while to try.

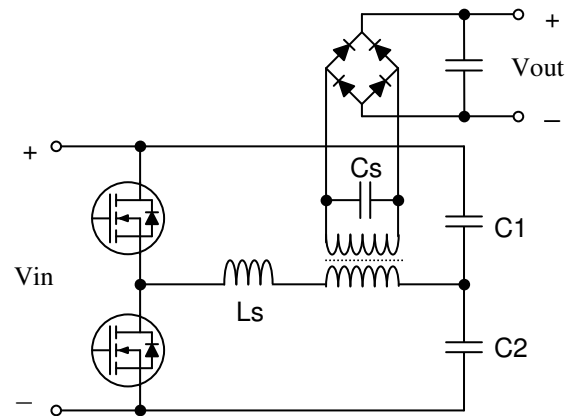


Figure 2 LCC

## The simple explanation

The LCC equivalent circuit is shown in figure 9. It looks like that for the LLC, just with an inductor replaced by a capacitor Cs.

It is evident that Ls and Cs form a 2<sup>nd</sup> order low pass filter. At a certain frequency Fo, the output of this filter is just high enough to touch ±Vo without rectification, so above Fo there is no power transfer. Going below Fo, there will be more or less power transfer from input to output.

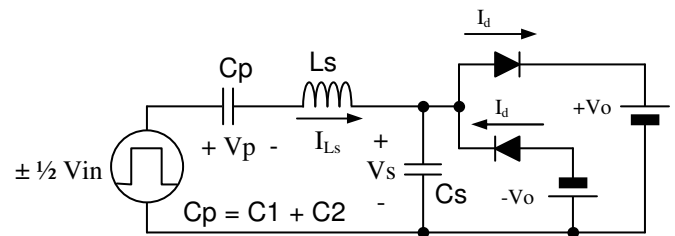


Figure 9 LCC equivalent circuit

But let's first have a look at an even simpler circuit: the "L" converter shown in figure 10 and its waveforms in figure 11. Cs has been removed and Cp is shorted,

In this simplified model all capacitors are gone, so it is will be misleading to name it a resonance converter. However this simple model still possesses the most basic properties and waveforms of the LCC resonance converter: the power delivered to the output reservoirs ±Vo will be inversely proportional to the frequency of the input generator. When the input voltage is a square wave with 1/2Vi > Vo, a current ILs will flow in the inductor as depicted, where the slopes of ILs will depend of the value of Ls: High Ls means low current slope steepness dILs/dt.

There will be a small delay between the input voltage steps and the zero crossings of ILs so the voltage Vs will be a delayed square wave. During the time between input and output steps, the voltage over the inductor is 1/2Vi + Vo, while in the remaining time it will only be 1/2Vi - Vo. Therefore, the slopes of ILs are different during these two intervals.

It is easy to see that if the frequency f is doubled, the peak currents will be only half. Since power P is equal to Vo • av(ILs), where av(ILs) is average sum of diode current, it can be shown that

$$P = \frac{(\frac{1}{2} \cdot V_{in})^2 - V_o^2}{8 \cdot f \cdot L_s} \cdot \frac{V_o}{\frac{1}{2} \cdot V_{in}}$$

Try to verify this equation. It is a good exercise.

Even though this simple converter cannot be named a resonance converter, it does illustrate the most basic mode of operation of the LCC converter. It is capable of regulating power by means of frequency. However in figure 10-11 zero power corresponds to an infinite frequency – not a nice feature in most cases. Further, if 1/2Vi ≤ Vo, no power at all can be transferred.

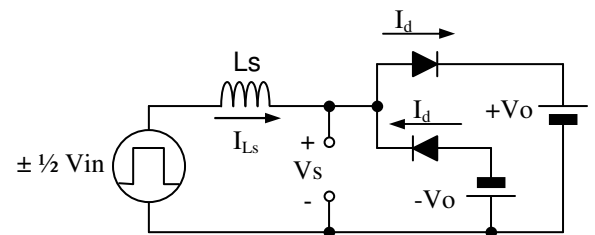


Figure 10 Simplified equivalent model

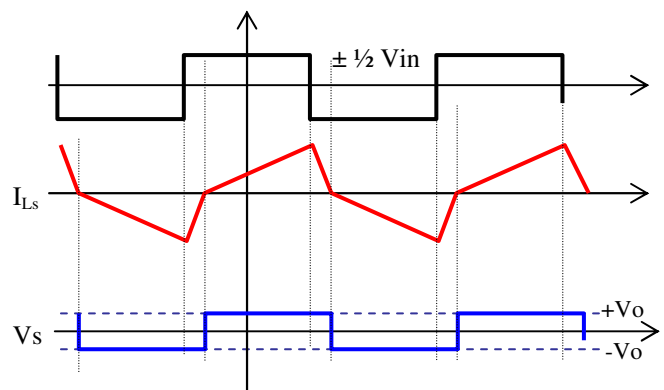


Figure 11 Simplified waveforms

**Behaviour of the LCC converter**

The LCC resonance converter model in figure 9 will behave like shown in figure 12-14, being in one of the three modes shown.

In figure 10 - 11 it was possible to establish a simple relation between control variable (frequency) and power. Unfortunately this is not possible in the circuit of figure 9.

Cs brings the zero-power frequency Fo down to a finite value, which can be handled by a control circuit. At frequencies above Fo the waveforms of the converter are relatively easy to predict: At this high frequency Cp can be considered a short circuit, and the converter becomes a simple LC low pass filter. The voltage Vs consists mainly of the first harmonic, whose amplitude is inversely proportional to frequency squared, and whose phase is 180 degrees delayed from the input square wave. Above Fo the amplitude of Vs is < Vo, so the rectifiers do not conduct – power is zero.

The slopes of ILS are shown as linear traces, but to be more accurate they are small fractions of long sine waves.

We will call this mode of operation “Mode 0”.

Now, as we let the frequency go down, we enter “mode 1” (figure 13) in which the amplitude of Vs passes the boundary at Vo. At this boundary, the diodes start to conduct small pulses of current, so the power is no longer zero. In the time intervals where the diodes conduct, Vs is clamped to ±Vo, and Vs looks less and less like a sine wave.

Figure 13 also indicates that in the diode conduction intervals, diode current = inductor current. This has to be so because Vs is flat, so current in Cs is zero during diode conduction.

Note that as diode current starts, Vs gradually shifts towards the left, i.e. the phase lag between input and the fundamental part of Vs becomes less than 180 degrees.

As frequency gets still lower, the diode current pulses get wider and higher, dominantly by shifting their leading edge to the left. At some frequency the start of diode conduction will pass the input transitions, and a third mode of operation is entered - “mode 2”.

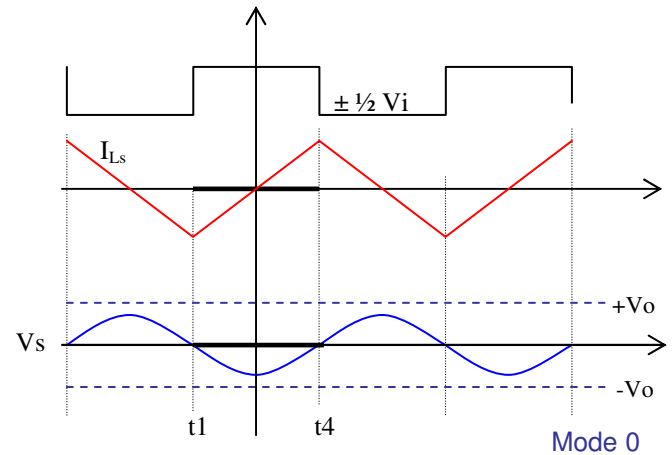


Figure 12

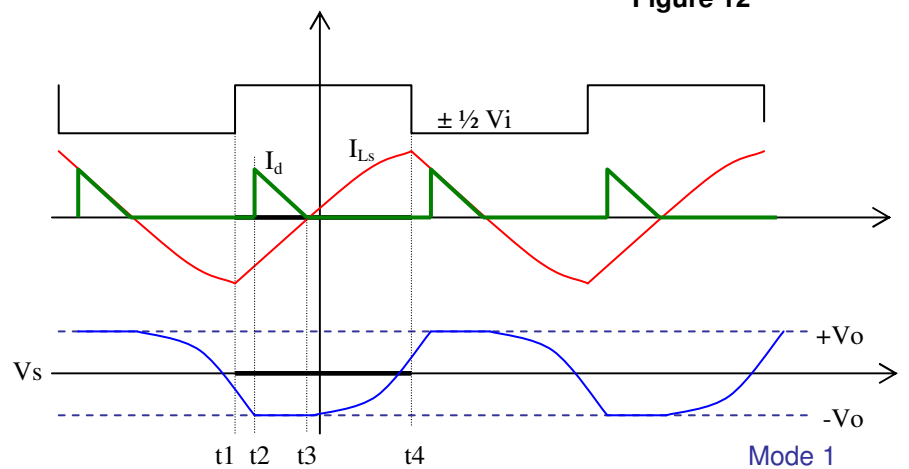


Figure 13

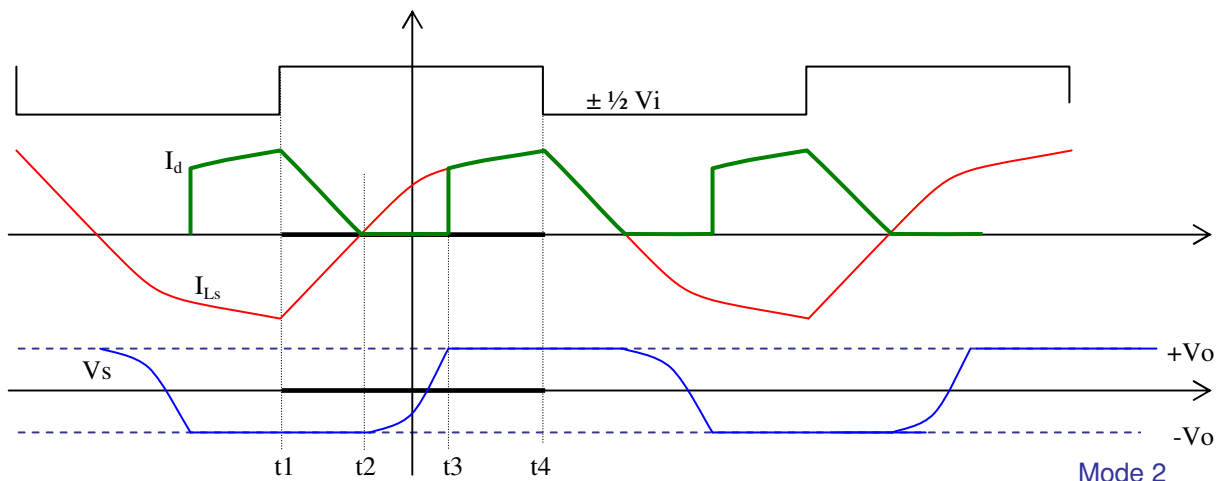


Figure 14

In mode 2 the diode current contains two distinct parts separated by the input transition. The fundamental part of  $V_s$  is shifted further to the left. The decrease of phase shift and increase of currents keep going on, as the frequency is lowered even more. When we approach the system's resonance frequency,  $V_s$  is nearly in phase with the input signal, and here a dramatic boost of currents and power suddenly occurs in a very narrow range of frequencies. A protection circuit must prevent operation very close to the resonance frequency where currents and voltages will otherwise destroy the converter.

Suppose we let the frequency jump below the resonance frequency, a fourth mode of operation "mode 3" would be entered, and indeed this mode can be used in a power converter. But since we want to operate above resonance, it does not make sense to go there. Should our LCC converter accidentally jump to mode 3, the control slope would be wrong: below resonance we would have positive feedback, where a lower frequency results in a lower power, and any feedback control system would then rush to the lowermost frequency and stay there for ever. And in mode 3 soft switching is lost which can be lethal for the fets.

The control circuit must prevent mode 3 from happening. We will discuss that in more detail later.

## Time Domain Approach for the LCC

The figures 12 - 14 tell in a qualitative way, what may happen in an LCC converter, when it is operated at various frequencies. Looking at the pictures, one can reasonably well realize that this is what the circuit in figure 9 will probably do. However it is extremely difficult for most people, including me, to puzzle out these pictures merely by brain power. This is true, despite the apparent simplicity of the circuit in figure 9. In fact the pictures were created after an intense study of the circuit by means of Pspice simulation.

At first, after realizing the above difficulties, I planned to use Pspice to help me design and optimize the LCC converter. But I was very disappointed to find that neither Pspice nor probably any other simulation tool can do this job. After one simulation run you have data only for one point in the curves, that should give you an overview of e.g. the output power as a function of frequency. Furthermore you may have to read the data from the simulation and plot them manually. Next, you would like to make plots of some important rms currents, core magnetization and maybe other interesting parameters as a function of power, and each of these curves would require you to read data from each individual simulation run and plot them manually.

When finished, of course you would like to study, what happens when you change the inductor value L. Or Cs. Or another component value or voltage.

It soon became clear that a better, faster, and much more direct tool was needed in order to make a fast and optimal design of an LCC converter. This realization resulted in my TDA analysis LCC tool in Mathcad back in 2001. Since then, of course, it has been improved and extended.

The common principle of the mathcad worksheet is the recognition that all currents and voltages at any time are fractions of sine or cosine wave shapes. This is true, assuming that there are no resistive elements in the circuit, i.e. no losses. The circuit is analysed mathematically only in one half period in which the input voltage is positive. The next half period must be identical but with opposite signs of voltages and currents.

Consider mode 1 (figure 13). The half period to be analysed must be split up in 3 sub-intervals :  $t_1 - t_2$  ,  $t_2 - t_3$  and  $t_3 - t_4$ . In the first and the last of these intervals no diodes are conducting, that means that only the resonance tank circuit L and ( $C_p$  in series with  $C_s$ ) is in action with the resonance angular frequency  $\omega_o = \frac{1}{\sqrt{L \cdot \frac{C_p \cdot C_s}{C_p + C_s}}}$

All what happens during these two intervals must obey this formula, i.e. all voltage and current wave shapes must be fractions of a sine or cosine with the period  $T_o = \frac{2 \cdot \pi}{\omega_o}$

In the middle interval a diode is conducting. That means that  $C_s$  is out of the game, so the wave shapes in the interval  $t_2 - t_3$  must obey the angular resonance frequency  $\omega_{oo} = \frac{1}{\sqrt{L \cdot C_p}}$  and  $T_{oo} = \frac{2 \cdot \pi}{\omega_{oo}}$ .

For most operating conditions  $T_o$  and  $T_{oo}$  are much longer than the actual switching period.

The worksheet investigates the circuit at one frequency at a time. The length of a half cycle ( $t_4 - t_1$ ) is defined from the start. The calculation starts with a guess for currents and voltages at time  $t_1$ , then  $t_2$  and the associated currents and voltages can be calculated from the sine and cosine relations which are valid in the first interval.

Next  $t_3$  is calculated in a similar way with its currents and voltages, and finally all currents and voltages at  $t_4$  are calculated. The final currents and voltages at  $t_4$  should be equal but opposite to the initial ones at  $t_1$ , but they are usually not, so an iteration is done with a new and better guess and so on, until initial and final currents and voltages are equal in size but opposite in sign. Now the worksheet knows all currents and voltages at this particular frequency, and they are stored in memory.

Next, the same iteration procedure is done at another lower frequency and so on, until mode 1 turns into mode 2. In mode 2 a new set of equations must be used, but the basic iteration method is the same.

Having passed through all three modes, the worksheet has stored data for all currents and voltages at as many frequencies as we like. Based on stored data it can now calculate peak currents, rms currents, core flux excursion and whatever we would like it to tell us. All these data are then plotted in a way that allows an easy overview of the characteristics with the selected set of component values.

It even has data to make a scope plot of currents and voltages (figure 15) to compare with simulation results or actual scope measurements.

The model in figure 9 implies a transformer with turns ratio 1:1. However the turns ratio should be used to adapt the model's output voltage  $V_o$  to the wanted output voltage  $V_{out}$ . The fictitious  $V_o$  must be selected relative to  $V_i$  such that the converter can supply power, typically  $V_o \approx \frac{1}{2} \cdot \min.V_i$ . Therefore, the turns ratio will usually have to be different from 1:1. The turns ratio can be simply calculated as "fictitious output voltage"  $V_o$  divided by "wanted output voltage"  $V_{out}$ . All calculated current data on the output side are multiplied by this ratio, and all output voltage data are divided by it.

Selected component values:

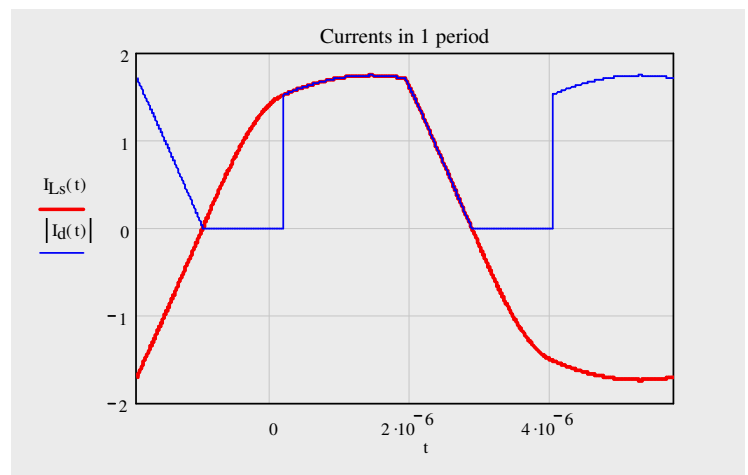
$C \equiv 2.15 \cdot 10^{-9}$      $L_s \equiv 200 \cdot 10^{-6}$      $C_s \equiv 3.92 \cdot 10^{-9}$

Fictitious output voltage (1:1 trafo):  $V_o \equiv 133$

Switching frequency:  $F \equiv 130 \cdot 10^3$

Input voltage:  $V_{in} \equiv 315$

Power = 130



$I_{L_s}(t)$ : current in inductor and primary switches.

$|I_d(t)|$ : current in output diodes all together (trafo = 1:1)

$V_i(t)$ : input square wave voltage.

$V_s(t)$ : voltage over secondary capacitor  $C_s$ . Will be clamped to  $\pm V_o$ .

$V_c(t)$ : voltage over primary series capacitor  $C$ .

$V_{prim}(t)$ : voltage over primary winding if  $L_s$  is the leakage inductance of the transformer.

$V_{L_s}(t)$ : Voltage over external inductor  $L_s$ .

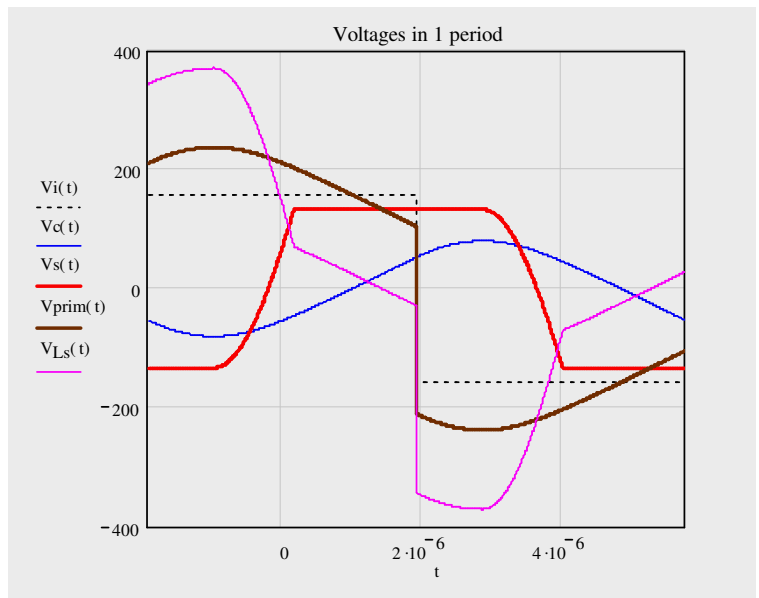
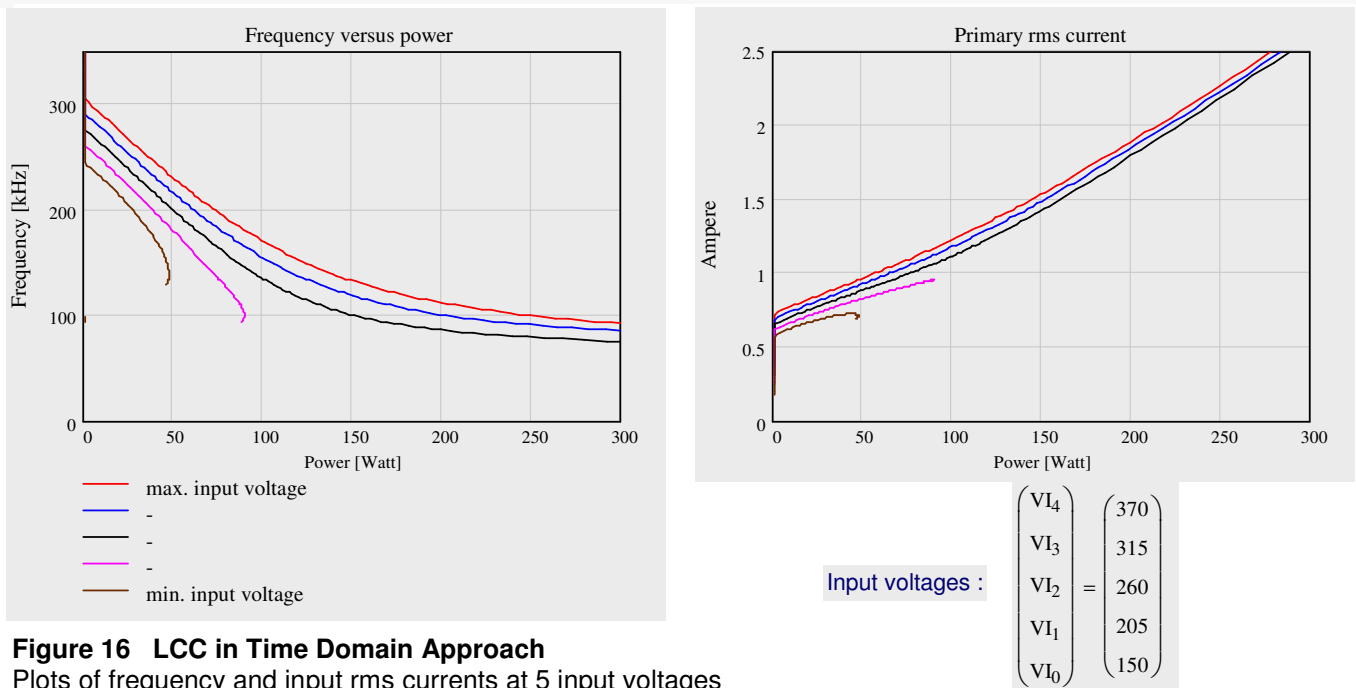


Figure 15 Calculated scope plots of currents and voltages

Two of the overview plots for the LCC converter specified in figure 15 are shown in figure 16. Compared to figure 7 for the LLC, this plot of switching frequency appears much different. Frequency is much more dependent on power. Frequency span must generally be larger for the LCC if we want to minimize no-load losses.



**Figure 16 LCC in Time Domain Approach**  
 Plots of frequency and input rms currents at 5 input voltages

For the curves with input voltage  $> 2 \cdot V_o$  ( $V_o = 133V$ ) the curves continue towards infinite power on the upper slope of a resonance peak.

A peculiar thing is that the LCC converter is able to transfer some limited power, even if input voltage  $< 2 \cdot V_o$ , which is seen from the two lower traces in figure 16. With the simplified converter in figure 10 – 11 this was not so. It may be explained by the LC-upswing created by  $L_s$  together with  $C_s$ . In some special cases this peculiarity can be a useful feature.

Another noticeable observation is that the plots are smooth and show no abrupt bends or jumps at the boundaries between mode 1 and mode 2. We can't even see where these boundaries are. This observation was done also for the LLC.

This math design tool has been my key to the LCC converter since 2001. It lets you analyse and see a converter's characteristics at a glance, and it lets you experiment with component values and get new results within seconds. And you can count on it - it is not an approximation 😊.

## Control issues

All half bridge resonance controller ICs contain a Voltage Controlled Oscillator. Typically, this oscillator's frequency is designed to be proportional to a control voltage or control current on a control pin, and the signal at the control pin is usually derived from the output error signal by linear amplification and filtering. This means that frequency is proportional to the error signal.

For instance, the VCO can be built like in L6598 or L6599 from ST (figure 17).

A control current is pulled out of pin 4. The oscillator capacitor is charged and discharged with a current proportional to the control current, thus generating a triangle voltage with fixed amplitude and variable frequency. The half bridge fets are turned on alternately on the up- and down-slopes of this triangle.

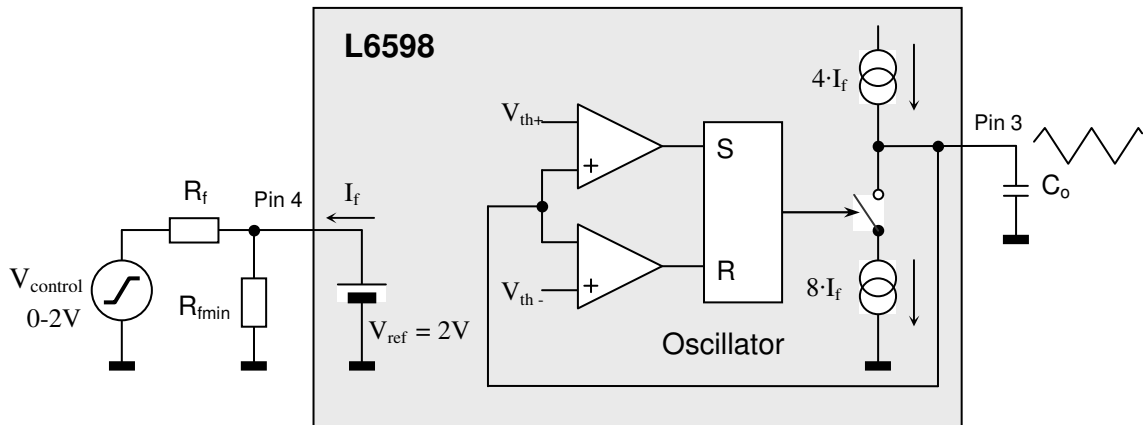


Figure 17 VCO example

For the same LCC converter as used in previous pages, the steady state transfer functions derived from figure 16 (left part) can look like figure 18. It is basically the same graph, just with x and y interchanged and x inverted. The steep part to the right hand side is the slope of a resonance top.

The slope of the curves is equivalent to steady state small signal gain. In this plot, steady state gain varies about 1:35 from 0W to 400W. In some converters it can be even more if we want to exploit the LCC's capability close to the resonance peak, for example for short peak loads.

When we close the feedback loop around this converter, the open loop gain will vary 35 times or more, when the output power goes from < 100W to > 400W. Everybody involved in feedback loop design knows, what this means: A feedback loop around this power stage will either be far too sluggish at low power or go into self oscillation at high power, or probably both. No attempt to linearize this loop by means of resistor-diode networks or similar will be very successful.

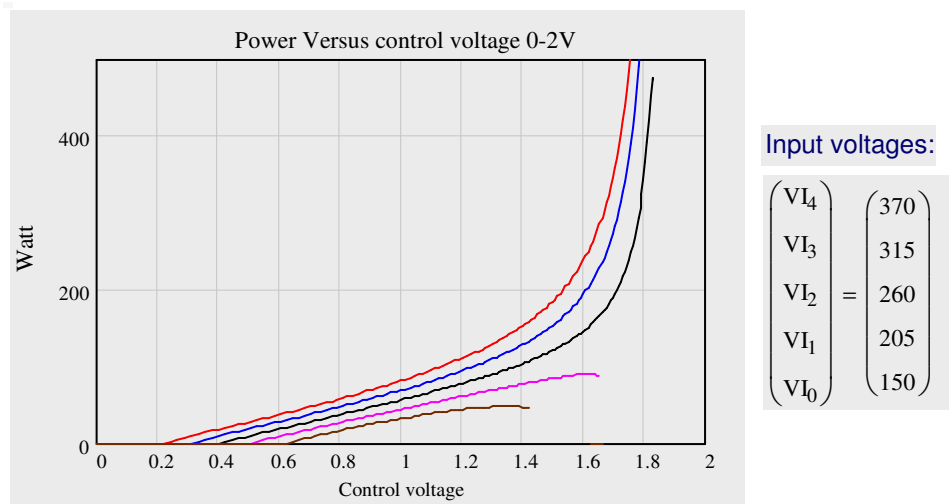


Figure 18 LCC power versus control voltage

There is another problem: close to the resonance peak the power stage will react with something like a 2<sup>nd</sup> order response to the control signal. When the control signal takes a step, the average current into the output capacitor will take a lot of cycles to move and settle at the new value, like in a mechanical swinging system. A 2<sup>nd</sup> order transfer function together with high gain will cause even more trouble.

Simply speaking : FORGET IT ! The LCC converter may be one of the world's best converters concerning efficiency and noise, but we cannot control it.

We could of course restrict the power range to max. 150W for this converter. But that would probably disqualify the LCC converter, compared to other topologies.

OK, let's see if the LLC is easier to control.

Figure 19 is one example of what the steady state transfer function could look like, corresponding to the left plot of figure 7. The horizontal position and spacing of the curves will depend on  $R_f$  and  $R_{fmin}$  (figure 17).

The middle curve is exactly at the Load Independent Point (400V input). The left hand curves are above the LIP where power is unlimited.

Apparently, the steady state transfer gain can be infinite and close to infinite, however above the LIP the curves have a finite but very variable slope.

In these calculated graphs the output voltage is assumed to be constant. This may not be quite true, unless the load is a battery. A large electrolytic output capacitor will also act like a battery at medium and high frequencies.

We should also remember that the curves show STEADY STATE

transfer function. The dynamic transfer function will have less gain because, like in the LCC at high power, it takes time to change the AC current in the LLC power stage. The situation could be compared to the buck converter in continuous current mode. It will also have an infinite steady state gain but its dynamic transfer function is a 2<sup>nd</sup> order low pass filter.

The problem with the LLC is that nobody can really tell much about its dynamic properties because they are incalculable. Most of the feedback loop design guides in application notes are oversimplified, some simply tell you to measure open loop gain and phase and then design your compensator to match the measurement.

The most comprehensive study of the LLC's dynamic transfer function is probably the PhD dissertation in ref. 7 which uses a vast amount of simulations to study the LLC's dynamic behaviour. Ref. 7 shows how single and double poles and zeros move around in a more or less chaotic way, depending on load and operating point. The results are also referred to in ref. 5. Simulated gains in the critical frequency range can vary tenfold or more with load and operating point. Typically, when gain is highest, phase lag is large due to the effect of a double pole.

Therefore, even if we could calculate the gain of the LLC converter, it would not make much sense. The calculations would probably tell us that an LLC converter with a wide load range and some input to output conversion range will be virtually uncontrollable.

You can be lucky to design an always stable LLC converter by using oversized output capacitors. But the design will be based on trial and error, and still, regulation can be extremely dependent on load and input voltage. I have seen more than one LLC converter which was self oscillating in some operating points, even a demo board from an IC supplier.

Unfortunately, we must conclude that, although both types of resonance converters have great potential in power conversion regarding efficiency and noise, their usefulness is severely limited due to huge gain variation and chaotically moving poles and zeros in the dynamic power transfer function.

But the resonance converters are really too promising to just accept that as a fact. And there is a way out.

It is indeed possible to turn the LCC and the LLC converters into a power stage with a constant, load independent gain and nearly immediate response – a controlled power source. Let's have a look in the next chapter.

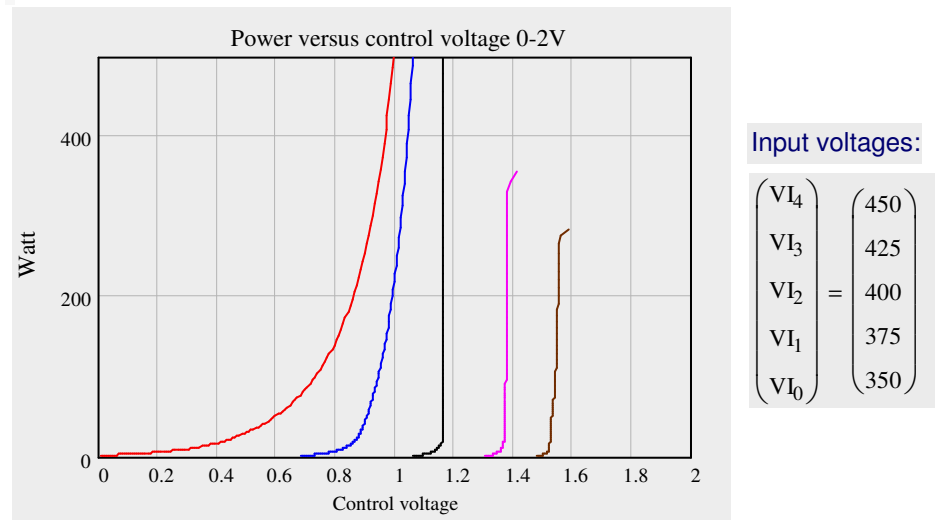


Figure 19 LLC power versus control voltage



## Charge Mode Control

An answer to the control obstacles in previous chapter is a new control method where you basically control charge, not frequency. It was found during my second LCC design where I was facing huge gain variations. In my first LCC design I managed to compensate the non-linearity reasonably well with a diode-resistor network. In the second design this was not enough. Having selected the LCC, because this design had to be a low noise design, there was no easy way back.

As so often before, when engineers are faced with a problem which has to be solved, they get crazy ideas. Experience tells me that nine out of ten good ideas are not good enough. This idea was number ten.

Together with the previously shown mathcad tools, this control method now enables me to design stable and fast responding LCC and LLC converters with a predictable and nearly linear feedback loop and a predictable step load response. In most designs I can now skip the breadboard and go directly into a prototype.

Charge Mode Control, as we named it, can be implemented quite easily but only in controllers with a triangle oscillator with external capacitor, like the L6598 or L6599 from ST. ICs with an oscillator at double frequency followed by a toggle flip-flop are not directly suitable. The trick is to inject a copy of the voltage on the resonance capacitor C, i.e. a copy of the charge flowing through the primary, into the oscillator, as shown in figure 20.

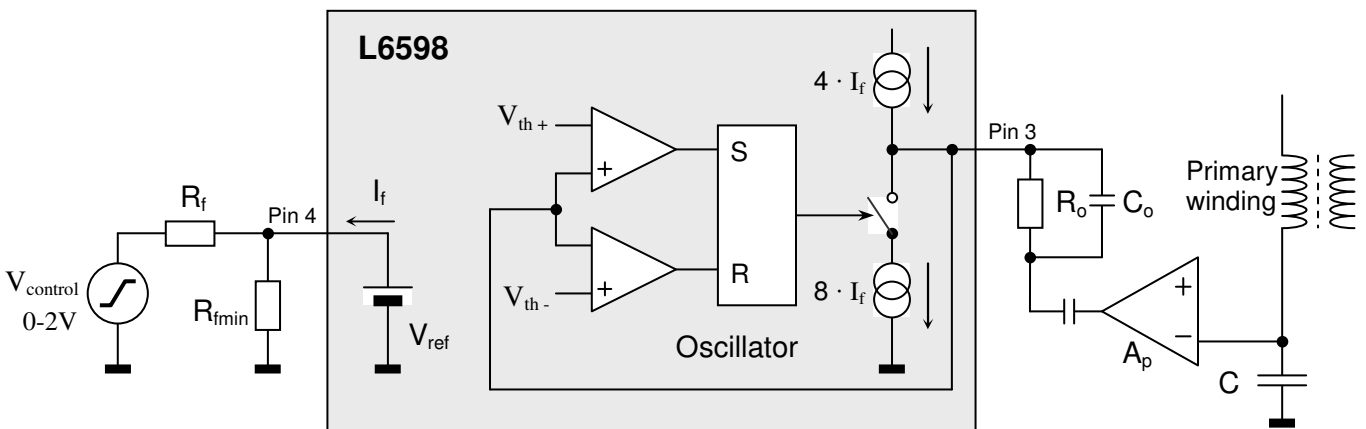


Figure 20 Charge mode model

The oscillator voltage at pin 3 now becomes the sum (or in this case the difference) of the original triangle voltage and a copy of the AC voltage on the resonance capacitor C. If  $R_o$  is present, the triangle will be distorted. By proper selection of  $R_o$ ,  $C_o$ , and the gain  $A_p$ , we can turn the curves in figure 18 and 19 into nearly linear curves with a finite slope. Figures 21 and 22 show what happens.

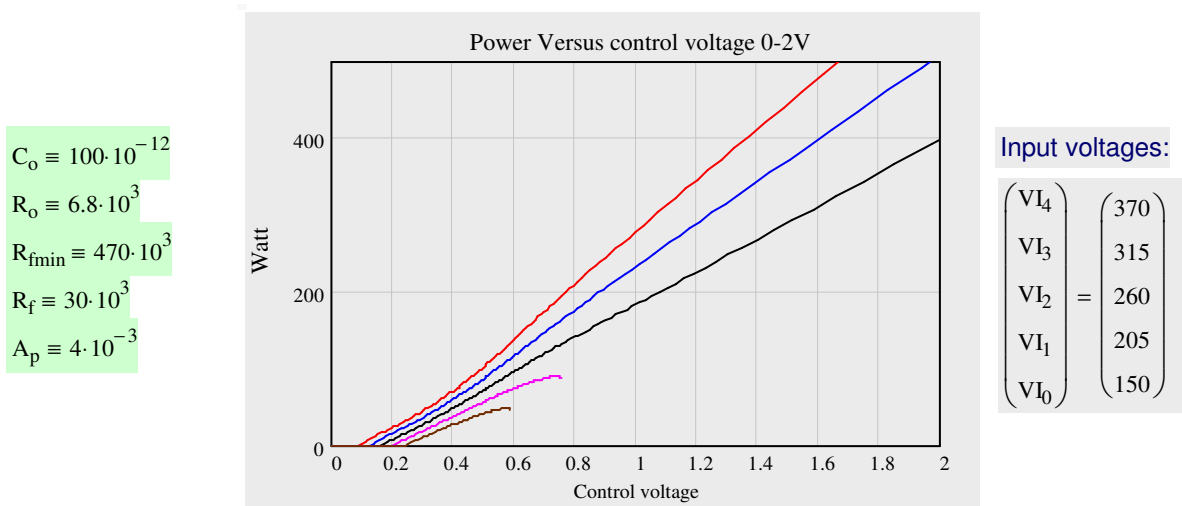


Figure 21 LCC with Charge Mode Control



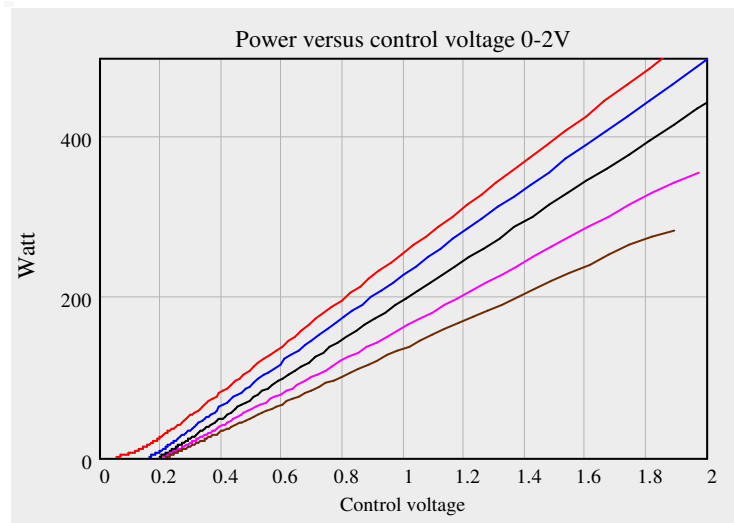
$$C_o \equiv 250 \cdot 10^{-12}$$

$$R_o \equiv 3 \cdot 10^3$$

$$R_{fmin} \equiv 50 \cdot 10^3$$

$$R_f \equiv 20 \cdot 10^3$$

$$A_p \equiv 2.2 \cdot 10^{-3}$$



Input voltages:

$$\begin{pmatrix} VI_4 \\ VI_3 \\ VI_2 \\ VI_1 \\ VI_0 \end{pmatrix} = \begin{pmatrix} 450 \\ 425 \\ 400 \\ 375 \\ 350 \end{pmatrix}$$

Figure 22 LLC with Charge Mode Control

Figure 21 and 22 display nearly linear control functions for the LCC as well as the LLC. However, the gain will vary proportional to input voltage, like we know it from a buck converter. In the LLC this proportionality can be exchanged with more parallel control lines with larger separation at low power, by leaving out  $R_o$ .

The LC “inertia effect”, causing a double pole at critical frequencies, is also gone. The power stage reacts immediately because we are controlling charge pr. cycle directly. The only effect we should take into account is a small “sampling” delay. For the LCC it is about  $\frac{1}{2}T$ , for the LLC it is about  $T$ , where  $T$  is switching cycle.

The slopes from figure 21 and 22 can be inserted as a gain factor in a feedback loop calculator, and the outcome of that fits extremely well with measurements on practical resonance converters.

Note that the curves are linear all the way from zero power to peak power. That is even better than the control of a buck converter where gain may change dramatically at the boundary between continuous and discontinuous current.

Charge Mode Control versus frequency control can be compared to the well known Current Mode Control versus Voltage Mode Control for pulse width modulated converters.

With voltage mode control, the currents in the system are not known by the regulation circuit. Only the on- and off-times of the switch are known. With current mode control it is different: peak current is the control variable, and the duty cycle is a secondary parameter which the control circuit does not care about. With current mode control we turn the 2<sup>nd</sup> order response of the power stage into a 1<sup>st</sup> order response which makes the design of a feedback loop simpler and less sensitive to tolerances.

With Charge Mode Control, the control circuit acts on charge pr. cycle and does not know or care about frequency. It turns a power stage with unknown and moving poles and zeros into a controlled power source with a finite and frequency independent gain.

Switchmode power supplies with large electrolytic output capacitors are often easy to stabilize because the low impedance of the bulk capacitor yields a low output impedance, even if loop gain is kept low. For LCC and LLC you would normally have to use “too large” capacitors to always maintain stability, and even then it can be difficult.

Charge Mode Control opens the door to using much lower output capacitors which means fast responding converters. One of my recent designs is an LCC converter for direct drive of a string of LEDs with either DC or several 100 Hz of PWM modulation. According to my math tools it should be no problem to control output current from zero to max. at this rate. Practical tests of the real converter confirmed this result. Without Charge Mode Control this would not have been possible.

The Charge Mode Control idea was patented by my customer: ref. 10.

## Sub-resonance protection.

For both the LLC and the LCC there is a maximum and often very limited power available at low input voltage, shown in figure 18 and 19 as the ends of the curves. The ends are exactly at the boundary to the hard switching area, i.e. on a more or less flat resonance top. Figure 18 and 19 tell us that if we want to exploit the high power capability at “normal” input voltage, the control circuit will want to push the frequency into the sub-resonance area at low input voltage. Many IC manufacturers try to solve the problem by specifying a very accurate frequency control to avoid going below the “resonance” frequency. But figure 18 and 19 clearly demonstrate that it will not work because the “resonance” top moves.

We need another method to prevent sub-resonance operation. For long time, I have used a discrete circuit which detects the proximity of current reversal and overrules the normal feedback path in that case, to prevent lower frequencies. This method is adaptive to the moving resonance top, it is smooth and non-latching, and it allows me to exploit the resonance converters to their full potential. Unfortunately, not many ICs contain that feature.

## Driver circuit.

Most resonance controllers have a built-in dead time between the high side and the low side drivers of a few hundred ns. This time is needed for the midpoint voltage to swing from zero to the positive rail and back. In the LCC the circulating current in the circuit is low at low load so the  $dV/dt$  is also low, and the transition time is high. It can easily be several 100 ns.

It is often necessary to insert  $dV/dt$  limiting snubber capacitors in parallel to each fet to achieve a low noise design. If the snubber capacitors become large, the transition time is

longer than the dead time at low load. This means that the snubber capacitors will be discharged some tens or a few hundreds of volts by the mosfets at each turn-on and it will cause high switching losses which may be enough to destroy the mosfets within some minutes – or anyway cause high losses at no-load.

Therefore, an IC with an adjustable dead time would be preferable, which would allow us to use a higher dead time in cases where an extremely low noise design is wanted. Figure 23 illustrates the minimum required dead time.

The driver circuit must create a fast turn-off, but the turn-on does not have to be fast, because the current in the mosfet runs backwards through its body diode after commutation. So there is plenty of time to turn the mosfets on. In fact, a higher dead time than the one set by the control ICs is usually wanted.

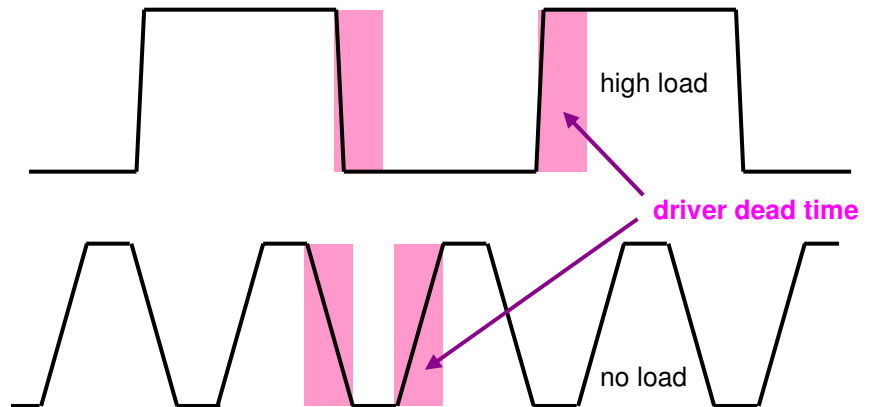


Figure 23 LCC waveforms at high and low load

## Summary of LCC and LLC properties.

After going through the properties and control issues of the LLC and the LCC converters, let us shortly list why and when it is a good idea to choose an LLC or an LCC resonance converter.

Some significant merits of the LLC and the LCC converter with Charge Mode Control are:

- No switching losses at all. Switching is soft at the input as well as at the output side. Even, or especially, when  $dV/dt$  snubber capacitors are used in parallel to the fets.
- Current stress of semiconductors is similar to the conventional half bridge forward converter.
- Voltage stress of mosfets is limited to input DC voltage.
- Voltage stress of output diodes is limited to output voltage. Nearly no overshoot or spikes.
- Therefore this topology invites to use low voltage diodes / schottkys with low forward losses.
- No need for dissipative snubbers.
- $L_s$  can be absorbed totally or partly as the leakage inductance of the transformer.
- In the LCC we make use of output winding capacitance and diode capacitance – especially interesting for high voltage outputs.
- The transformer can be constructed with low capacitance between primary and secondary, because no good magnetic coupling is required. This yields a low common mode noise.
- Potentially very low noise level, especially in the megahertz range.
- Capable of high peak power.
- In the LCC, a rectangular voltage / current output characteristic is easy to obtain – nice overload and short circuit protection.
- It is easy to implement synchronous rectifiers on the output because diodes turn off with low  $dI/dt$ .
- The constant current limit function can easily be supplemented by a constant power limit (if input is fixed).
- Static and dynamic transfer function can be nearly linear all the way from zero power to max. peak power.
- No signs of instability or “pulse skipping” at no load, as seen with pulse width modulated topologies.
- No continuous / discontinuous current boundary as in pulse width modulated topologies.
- Most control ICs contain a built-in high side driver for the upper mosfet.
- Experience has shown that the LCC and LLC converters can be extremely robust and reliable.

Of course it's not all joy and happiness. There are a few negative sides too:

- Frequency is variable. In some applications that is not allowed.
- Not suitable for low output voltage with high current due to output ripple.
- Output DC capacitor must absorb a much higher ripple current than in the forward converters. (but still only about half of the ripple current in the output of an equivalent flyback converter).
- LCC: A wide input voltage range is possible, but unlike the half bridge forward converter, the LCC converter must then suffer from high rms currents over the whole input voltage range.
- LLC: Only suitable for a narrow input and output voltage range – at full power at least.
- At no load there must still be currents flowing in the circuit – some watts will usually be consumed at no load (unless a burst mode is implemented).
- Adjustment of output voltage to zero is not possible at no load. Zero output voltage requires some load current.
- Resonance capacitors must withstand high AC currents and must be special types.
- LCC: High switching frequency (up to for instance 400kHz) at low load may cause EMI trouble.
- The most severe drawback is maybe the difficulty in designing and optimizing an LLC, or especially an LCC converter. Nobody can be sure to hit a good design by intuition or simple hand-math. This is where the described math tools can help you.

## LLC or LCC?

Having fast and accurate tools for both LLC and LCC, a power supply specification can now be evaluated on my computer for both types, and consequently the best one can be chosen in each case.

Today, I do about 50% of each type. LLC have clear advantages if input voltage (and output voltage) are reasonably constant. For instance if there is a Power Factor Correction stage in front and output voltage is fixed, the LLC can make a wonderful compact solution with splendid efficiency and low noise. It is always the PFC stage which causes noise trouble.

In applications where input voltage varies 2:1 or more, or where output voltage is variable (battery chargers, LED current sources etc.) the LLC carries a lot of design trouble with it. In such cases the LCC usually appears to be more friendly.

An exception to this statement is applications where you want the power to be somehow proportional to input voltage in a wide input range. The LLC seems to be tailor made for this application, or at least better than the LCC.

It is still a mystery to me why only the LLC converter has attracted the world's attention.

For the LCC you can use quite simple reasoning to prove that it should work, like the discussion around figure 10 – 14. The LLC turns out to be several times more difficult to describe than the LCC. The LLC will move between 6 useful modes, compared to only 2 modes for the LCC. I always wonder how anyone ever got the idea for the LLC because it is absolutely not intuitive to me that it should work.

As mentioned, the LCC wins the battle in my computer if the input or output voltage range is 2:1 or more. It is also the preferred choice at very high output voltages because it has a capacitor on the output winding. At high voltage, the parasitic capacitances in windings and diodes normally cause trouble with ringings and additional power loss etc. In the LCC, the parasitic capacitances are absorbed in the output AC capacitor.

## The ideal control IC

In my resonance converter designs up to now, all the described features are built in plus an additional non-latching current limiter. Hence the LLC and the LCC converters will beat the traditional buck derived half bridge, push-pull, or dual switch forward at nearly any time. Even the design time for the resonance converters is shorter.

But most of the features are add-on to the L6598 by means of discrete circuits. They are very cheap of course but the component count is much higher than it could be.

It should be evident from the previous discussion that a better and more intelligent resonance controller IC would be very welcome, featuring things like Charge Mode Control, current limit (preferably non-latching), sub-resonance protection (non-latching), adjustable dead time, 600V high/low side mosfet driver. For low standby-power designs, also a working and reliable noise free burst mode should be contained in it. The simple implementation of burst mode in some of to-day's ICs does not work well.

*To the best of my belief, neither the IC manufacturers nor the SMPS designers are generally aware of all the design constraints and pitfalls – and the potential great advantages – in resonance converters. Some designers have tried and burnt their fingers fiercely. This is not strange at all, because resonance converters is one of the toughest areas in SMPS design.*

*The described tools and control method in this paper is my contribution to change that situation.*

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Inventors: NIELSEN RUNO (DK); CHRISTENSEN SOEREN KJAERULFF (DK).  
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Ref. 6 deals with the analysis of the LCC converter, apparently with methods very similar to mine. However, the results are not immediately usable or verifiable, and the results were not translated into a practical design tool.